# Hermite Foveation

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## Abstract

A foveated image is a non-uniform resolution image whose resolution is highest at a point (fovea), but falls off away from the fovea. In this paper we shall consider foveation, based on new projection scheme of image preprocessing. It is based on an expansion into series of eigenfunctions of the Fourier transform.

***Keywords:*** *Foveation, Fourier transform, Hermite functions, image processing.*

## 1. INTRODUCTION

Foveation refers to the creation and display of the signal where the resolution varies across the signal. The highest resolution region is called the foveation region. Usually foveation is used in image processing [1]. The primary value of foveation is in compression [2], [3] and forced focusing. Image foveation exploits the fact that the resolution of the human visual system declines away from the direction of gaze, so it is only necessary to transmit fine detail in the direction of gaze.

Image foveation can also be used in vision research [4]. For example, it can be used in combination with eye tracking to precisely control the spatial information available across the retina of the eye. This makes image foveation a potentially valuable tool for analyzing the contributions of different retinal regions to task performance.

The aim of the work is Hermite foveation and its application to image processing. The proposed method is based on the features of Hermite functions and properties of foveation. The Hermite functions are widely used in pure mathematics [5] and image processing [6], [7]. It is also necessary to underline that the joint localization of Hermite functions in the both frequency and temporal spaces makes using these functions very stable to information errors.

## 2. THE ALGORITHM

In common case the foveation operator $T$ of a function $f(x)$ can be treated as an integral operator [8]:

$$(Tf)(x) = \int_{-\infty}^{\infty} k(x,t) f(t) dt$$

where $k(x,t)$ is the kernel of the operator:

$$k(x,t) = \frac{1}{\alpha|x-\gamma| + \beta} g\left( \frac{t-x}{\alpha|x-\gamma|+\beta} \right)$$

and

$\alpha$ – relative foveation speed;

$\gamma$ – the foveation point (fovea);

$\beta$ – the resolution in the foveation area;

$g(x)$ – the smoothing function.

An example of the foveation is depicted below:

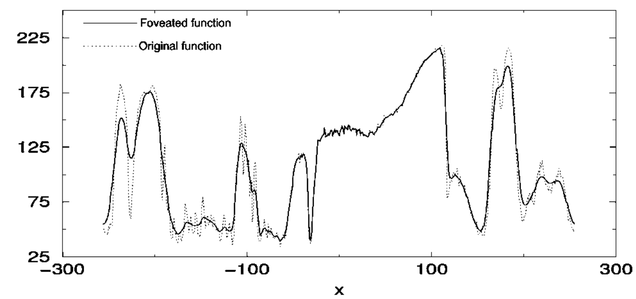


**Figure 1:** Foveation [8]

For foveation of the signal $[0, w-1]$ in the point $\gamma$ by $n$ Hermite functions $\psi_i(x)$, which are localized on $[-A_i, A_i]$, in $K$ steps we define kernel $k(x,t)$ as:

$$k(x,t) = \sum_{i=0}^{\frac{n}{K}-1} \psi_i\left( A_{\frac{n}{K}-1} \frac{2x-w+1}{w} \right) \psi_i\left( A_{\frac{n}{K}-1} \frac{2t-w+1}{w} \right) + \\ + \sum_{j=1}^{K-1} \left( \begin{array}{l} \max\left( \min\left( \frac{r}{r-1}\left(1 - \frac{2r^j|\gamma-x|}{w}\right), 1\right), 0\right) \cdot \\ \sum_{i=0}^{\frac{n}{K}-1} \psi_i\left( A_{\frac{n}{K}-1} \frac{2x-w+1}{wr^j} \right) \psi_i\left( A_{\frac{n}{K}-1} \frac{2t-w+1}{wr^j} \right) \end{array} \right)$$

where:

– a number of foveation steps (lays) $K \geq 2$,

– decreasing coefficient for foveation area between lays $r > 1.0$,

– a number of Hermite functions $n$ must satisfy an inequality

$K \leq n \leq 750K$

(high bound is connected to the precision of the 64 bits double data),

– a fovea $\gamma \in [0, w-1]$,

– a foveation area is

$$\left[ \max\left(0, \gamma - \frac{w}{2r^{K-1}}\right), \min\left(w-1, \gamma + \frac{w}{2r^{K-1}}\right) \right],$$

– a resolution in the foveation area is $\beta = \frac{n}{K}$,

– the Hermite functions $\psi_n(x)$ are defined as:

$$\psi_n(x) = \frac{(-1)^n e^{x^2/2}}{\sqrt{2^n n! \sqrt{\pi}}} \cdot \frac{d^n(e^{-x^2})}{dx^n}$$

in one-dimensional case and

$$\psi_{nm}(x,y) = \frac{(-1)^{n+m} e^{x^2/2+y^2/2}}{\sqrt{2^{n+m} n! m! \pi}} \cdot \frac{d^n(e^{-x^2})}{dx^n} \cdot \frac{d^m(e^{-y^2})}{dy^m}$$

in two-dimensional case.

The scheme of foveation-coding, based on described algorithm, is depicted below:

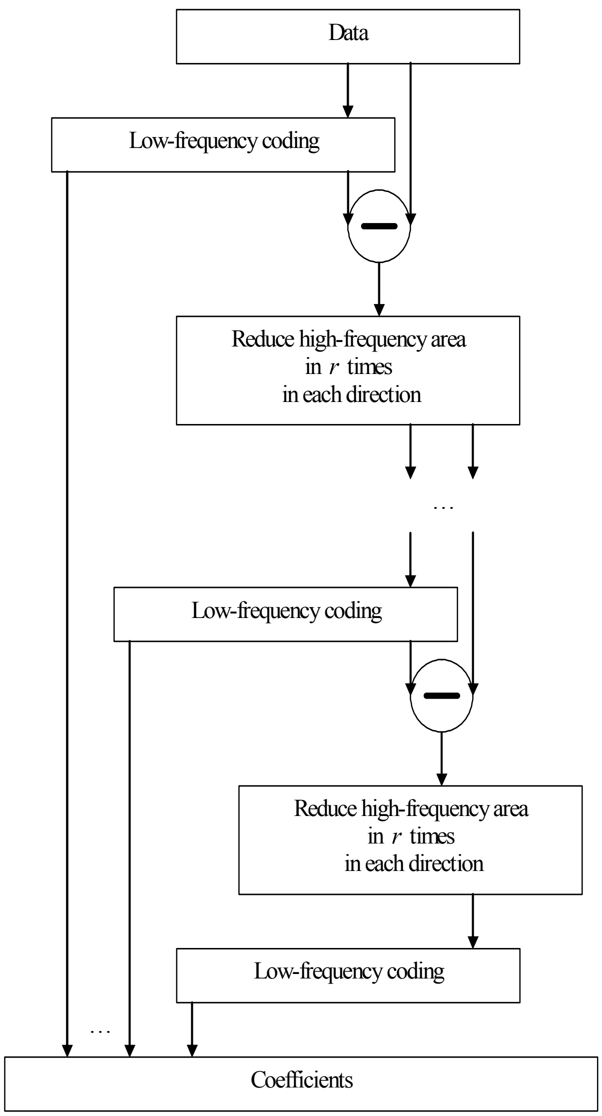


**Figure 2:** Scheme of foveation-coding

The scheme of foveation-decoding is depicted below:

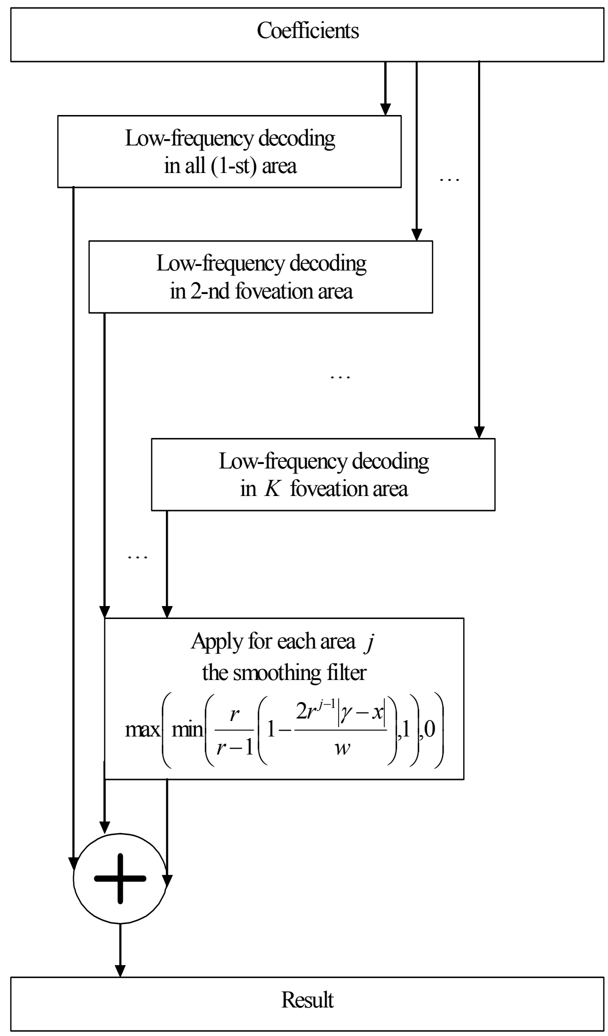


**Figure 3:** Scheme of foveation-decoding

An example of the 4-lays foveation by 128 Hermite functions of the signal $[0,511]$ in the point $\gamma = 265$ with $r = 2.0$ is depicted below:

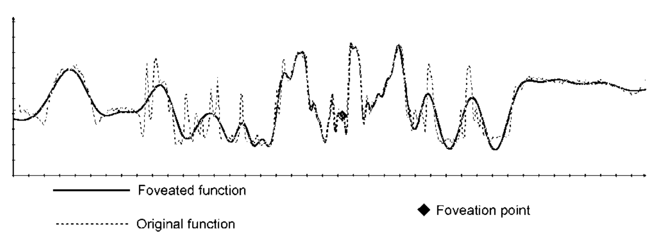


**Figure 4:** Hermite foveation

## 3. IMAGE FOVEATION

For image Hermite foveation we divide original image to three color 2D arrays (red, green, blue). After that we applied sequence of 2D Hermite transforms around fovea with gradually decreasing area of foveation. Inside decoding we apply additional smoothing filter for deletion borders between foveation areas on different lays.

Let's consider density changing, depending on different parameters, and schemes of coefficients location ($K$ - a number of foveation steps (lays), $r$ - a decreasing coefficient for foveation area between lays, $\gamma$ - fovea, $c$ - center of an image):

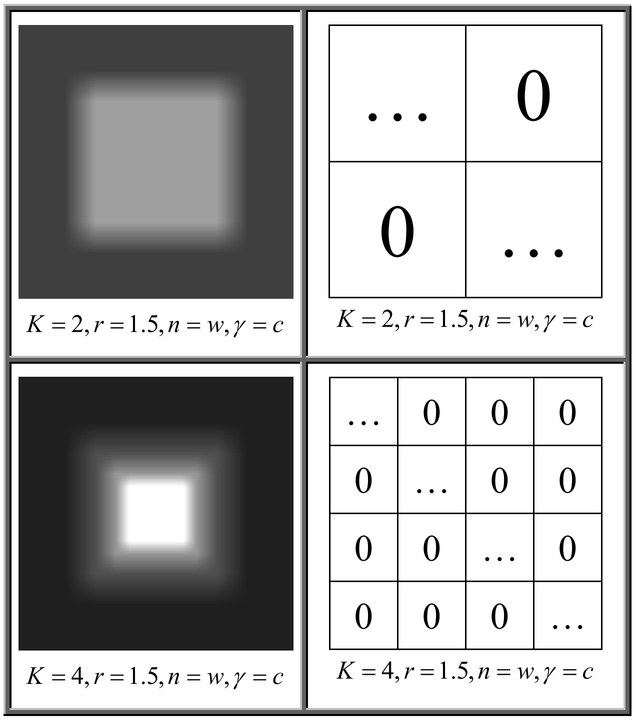


**Figure 5:** Foveation density and coefficients location

So, using Hermite foveation with $K$ lays we compress useful information $K$ times. Moreover, the performance of coding/decoding is improved approximately $K$ times also.

For comparison we have chosen popular image called "Lena" with size 512x512 pixels. Other popular methods of foveation, based on wavelet foveation [8], are depicted below with results, received from our method. Fovea $\gamma$ in each case is located in right eye.

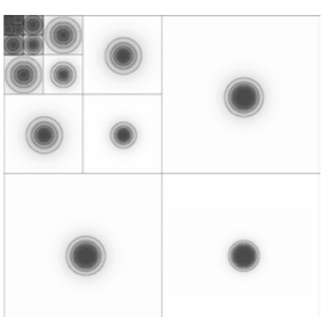

**Figure 6:** Mask $\alpha$

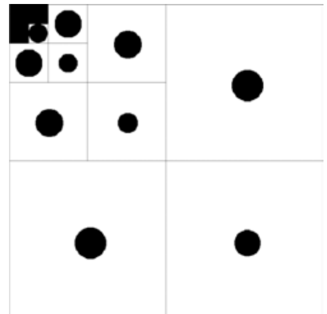

**Figure 7:** Mask $\beta$

**Figure 8:** Original image (512x512 200dpi)

**Figure 9:** Wavelet foveation with mask $\alpha$

**Figure 10:** Wavelet foveation with mask $\beta$

**Figure 11:** Hermite foveation
$K = 4, r = 1.5, n = 512$

**Figure 12:** Hermite foveation
$K = 8, r = 1.3, n = 512$

## 4. CONCLUSION

In this paper we considered the task of Hermite foveation. It is based on the features of Hermite functions and common foveation operator properties. We have used an expansion into series of eigenfunctions of the Fourier transform, which has enabled us using advantages of a time-frequency analysis. On the other hand Hermite foveation allows us to compress useful data and to improve performance of coding/decoding. So we can conclude that proposed method is comparable with wavelet foveation.


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